Markscheme

November 2020

Further mathematics

Higher level

Paper 1

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 ***N*** marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

  $$f'(x) = \big(2\cos(5x-3)\big)5 \quad \big(=10\cos(5x-3)\big)$$ ***A1***

  Award ***A1*** for $\big(2\cos(5x-3)\big)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 1, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Further Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. using l'Hôpital's rule,

$\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}=\lim_{x\to 0}\frac{2\cos x-2\cos 2x}{3x^2}$ ***M1A1***

$=\lim_{x\to 0}\frac{-2\sin x+4\sin 2x}{6x}$ ***(M1)A1***

$=\lim_{x\to 0}\frac{-2\cos x+8\cos 2x}{6}$ ***A1***

$=1$ ***A1***

[6 marks]

2. (a) **METHOD 1**

sum of degrees of vertices $=3+5+5+5+4+4=26$ ***A1***

number of edges $e=13$ ***A1***

the sum is equal to twice the number of edges which verifies the handshaking lemma ***R1***

METHOD 2

degrees of vertices $=3,5,5,5,4,4$ ***A1***

there are 4 vertices of odd order ***A1***

there is an even number of vertices of odd order which verifies the handshaking lemma ***R1***

[3 marks]

(b) if planar then $e\le 3v-6$ ***M1***

$e=13,\ v=6$ ***A1***

inequality not satisfied ***R1***

therefore G is not planar ***AG***

Note: method explaining that the graph contains $\kappa_{3,3}$ is acceptable.

[3 marks]

(c) there are vertices of odd degree ***R1***

hence it does not contain an Eulerian circuit ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [8 marks]

3. (a) each row and each column contain every element (exactly once) ***A1***

[1 mark]

(b) the identity element is c ***A1***

[1 mark]

(c) all elements are self-inverse ***A1***

[1 mark]

(d) (i) $a*(b*d)=a*e$ *(M1)*

$=d$ ***A1***

(ii) $(a*b)*d=e*d$

$=b$ ***A1***

[3 marks]

(e) because it is not associative ***R1***

$\{S,*\}$ is not a group ***A1***

Note: Do not award ***R0A1***.

Note: Follow through from (d)

[2 marks]

Total [8 marks]

4. (a) $\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}=0$ ***M1***

$(a-\lambda)(d-\lambda)-bc=0$ ***M1A1***

$\lambda^2-(a+d)\lambda+ad-bc=0$ ***A1***

$\alpha=(a+d);\beta=ad-bc$

[4 marks]

(b) (i) $\boldsymbol{A}^2=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$ ***(M1)A1***

$\boldsymbol{A}^2-(a+d)\boldsymbol{A}+(ad-bc)\boldsymbol{I}=$

$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}-(a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ***M1***

$=\begin{bmatrix} a^2+bc-a(a+d)+ad-bc & ab+bd-b(a+d) \\ ac+cd-c(a+d) & bc+d^2-d(a+d)+ad-bc \end{bmatrix}$ ***A2***

$=\boldsymbol{0}$ ***AG***

Note: Award ***A1A0*** for a single error.

(ii) multiply throughout by $\boldsymbol{A}^{-1}$ giving ***M1***

$\boldsymbol{A}-\alpha\boldsymbol{I}+\beta\boldsymbol{A}^{-1}=\boldsymbol{0}$ ***A1***

$\boldsymbol{A}^{-1}=\frac{1}{\beta}(\alpha\boldsymbol{I}-\boldsymbol{A})$ ***AG***

[7 marks]

Total [11 marks]

5. (a) let X have probability density function f. Then

$$\mathrm{E}(X) = \int_a^b x f(x)\mathrm{d}x \left(= \int_a^b x \frac{\mathrm{d}F(x)}{\mathrm{d}x}\mathrm{d}x \right)$$ ***M1***

$$= \left[xF(x)\right]_a^b - \int_a^b F(x)\mathrm{d}x$$ ***M1A1***

$$= b \times 1 - a \times 0 - \int_a^b F(x)\mathrm{d}x$$ ***A1***

$$= b - \int_a^b F(x)\mathrm{d}x$$ ***AG***

[4 marks]

(b) $\mathrm{E}(X) = \frac{\pi}{4} - \int_0^{\frac{\pi}{4}} \tan x \mathrm{d}x$ ***M1***

$= 0.439$ ***A1***

[2 marks]

(c) the median m satisfies $F(m) = \tan m = 0.5$ ***M1***

$m = 0.464$ ***A1***

[2 marks]

Total [8 marks]

6. attempting to use theoretical method $x \equiv a^{p-2}b \pmod{p}$ ***(M1)***

$x \equiv 5^{11-2} \times 4 \pmod{11} \equiv 3 \pmod{11}$ ***(A1)***

general solution is $3, 14, 25, 36, 47, \ldots$ ***(A1)***

for the second equation

$x \equiv 11^{7-2} \times 6 \pmod{7} \equiv 5 \pmod{7}$ ***(A1)***

general solution is $5, 12, 19, 26, 33, 40, 47, \ldots$ ***(A1)***

the smallest positive value of x satisfying both congruences is 47 ***A1***

Note: Accept use of Chinese remainder theorem, using a table, substitution of one congruence into another. Add methods at standardisation.

[6 marks]

7. (a) a stretch of scale factor 3 in the x direction
and a stretch of scale factor 2 in the y direction ***A1***

[1 mark]

(b) (i) the four sides are equal in length (5) ***A1***

$\text{Grad AB} = \frac{4}{3}$, $\text{Grad BC} = -\frac{3}{4}$ ***A1***

so product of gradients $= -1$, therefore AB is perpendicular to BC ***A1***

therefore ABCD is a square ***AG***

(ii) area of square $= 25$ ***A1***

(iii) the transformed points are

$A' = (3,8)$

$B' = (12,16)$

$C' = (24,10)$

$D' = (15,2)$ ***A2***

Note: Award ***A1*** if one point is incorrect.

(iv) $\text{Grad } A'B' = \frac{8}{9}$; $\text{Grad } C'D' = \frac{8}{9}$ ***A1***

therefore $A'B'$ is parallel to $C'D'$ ***R1***

$\text{Grad } A'D' = -\frac{6}{12}$; $\text{Grad } B'C' = -\frac{6}{12}$ ***A1***

therefore $A'D'$ is parallel to $B'C'$

therefore $A'B'C'D'$ is a parallelogram ***AG***

(v) area of parallelogram = |determinant| × area of square (***M1***)

$= 6 \times 25$

$= 150$ ***A1***

[11 marks]

Total [12 marks]

8. (a) (2,7) (3,9) (4,10) (5,8) (6,11) ***A4***

Note: Award ***A3*** for 4 correct, ***A2*** for 3 correct, ***A1*** for 2 correct.

[4 marks]

(b) (i) $\{\{1,12\},\times_{13}\}$ ***A1***

(ii) $\{\{1,3,9\},\times_{13}\}$ ***A2***

Note: Accept just the sets.

[3 marks]

(c) (i) coset of 2 = {2,10,3,11} ***(M1)A1***
coset of 3 = {3,2,11,10} ***A1***
coset of 4 = {4,7,6,9} ***A1***

(ii) cosets of different elements are either identical or disjoint ***A1***

[5 marks]

Total [12 marks]

9. (a) (i) $G_x(t) = p + pqt + pq^2t^2 + \ldots$ ***M1A1***

this is an infinite geometric series with first term p and common ratio qt, so (provided $|qt| < 1$) ***R1***

$G_x(t) = \dfrac{p}{1-qt}$ ***AG***

(ii) $G_x'(t) = \dfrac{pq}{(1-qt)^2}$ ***A1***

$G_x''(t) = \dfrac{2pq^2}{(1-qt)^3}$ ***A1***

use of $\mathrm{Var}(X) = G_x'(1) + G_x''(1) - [G_x'(1)]^2$ ***M1***

$\mathrm{Var}(X) = \dfrac{pq}{(1-q)^2} + \dfrac{2pq^2}{(1-q)^3} - \dfrac{p^2q^2}{(1-q)^4}$ ***A1***

$= \dfrac{1-p}{p} + \dfrac{2(1-p)^2}{p^2} - \dfrac{(1-p)^2}{p^2}$ ***A1***

$= \dfrac{1-p}{p^2}$ ***A1***

[9 marks]

(b) (i) $G_y(t) = \left(\dfrac{p}{1-qt}\right)^4$ ***A1***

continued…

Question 9 continued

(ii) **METHOD 1**

$\mathrm{P}(Y=3) =$ coefficient of t^3 in the expansion of $G_y(t)$ ***(M1)***

$= p^4 \times \dfrac{4\times5\times6}{3!}\times q^3$ ***A1***

$= 20p^4(1-p)^3$ ***A1***

METHOD 2

$\mathrm{P}(Y=3) = \dfrac{G_y'''(0)}{3!}$ ***(M1)***

$G_y'''(t) = \dfrac{120p^4q^3}{(1-qt)^7}$ ***A1***

$\mathrm{P}(Y=3) = 20p^4(1-p)^3$ ***A1***

[4 marks]

Total [13 marks]

10. (a)

$$\begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 2 & 3 \\ -1 & 4 & 0 & 5 \\ 1 & 7 & 1 & 9 \end{bmatrix}$$

using row operations, the matrix becomes for example

$$\begin{bmatrix} 1 & 0 & \frac{4}{11} & \frac{1}{11} \\ 0 & 1 & \frac{1}{11} & \frac{14}{11} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ ***(M1)A1***

this reduced matrix contains 2 independent rows so the rank of $\boldsymbol{M}$ is 2 ***A1***

Note: Allow the use of ref on GDC.

[3 marks]

(b) in order to span the space of 4-D column vectors, the 4 vectors in S would have to be independent so the rank of $\boldsymbol{M}$ would have to be 4 which it is not ***R1***

[1 mark]

(c)

let $\begin{bmatrix} 7 \\ 12 \\ 2 \\ 9 \end{bmatrix} = a\begin{bmatrix} 2 \\ 5 \\ -1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 3 \\ 2 \\ 4 \\ 7 \end{bmatrix} + c\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + d\begin{bmatrix} 4 \\ 3 \\ 5 \\ 9 \end{bmatrix}$ ***M1***

Note: Accept two independent columns.

Note: Allow the use of ref on GDC.

attempting to solve on GDC, we find that there is an infinite number of solutions ***R1***

therefore the given vector does belong to the subspace spanned by S ***A1***

[3 marks]

Total [7 marks]

11. (a) consider $\int_2^N \frac{\ln x}{x^2}\mathrm{d}x$ ***M1***

attempt to integrate by parts ***M1***

$= \left[-\frac{1}{x}\ln x\right]_2^N + \int_2^N \frac{1}{x^2}\mathrm{d}x$ ***A1***

$= \left[-\frac{1}{x}\ln x\right]_2^N - \left[\frac{1}{x}\right]_2^N$ ***A1***

Note: Condone absence of or wrong limits up until this point.

$= \frac{1}{2}\ln 2 - \frac{1}{N}\ln N + \frac{1}{2} - \frac{1}{N}$ ***A1***

as $N \to \infty$, $\frac{\ln N}{N} \to 0$ (using l'Hôpital's rule once) and $\frac{1}{N} \to 0$ ***A1***

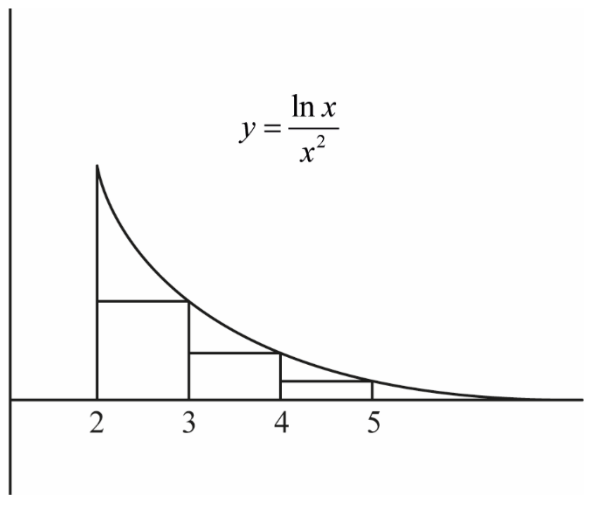
therefore

$\int_2^\infty \frac{\ln x}{x^2} = \frac{1}{2}(1+\ln 2)$ ***A1***

the given series is therefore convergent because this integral is convergent ***R1***

[8 marks]

(b) (i)



A1

Note: Accept the absence of rectangles.

(ii) from the diagram,

$$\sum_{n=3}^{\infty} \frac{\ln n}{n^2} < \int_2^{\infty} \frac{\ln x}{x^2} \mathrm{d}x = \frac{1}{2}(1 + \ln 2)$$ ***M1A1***

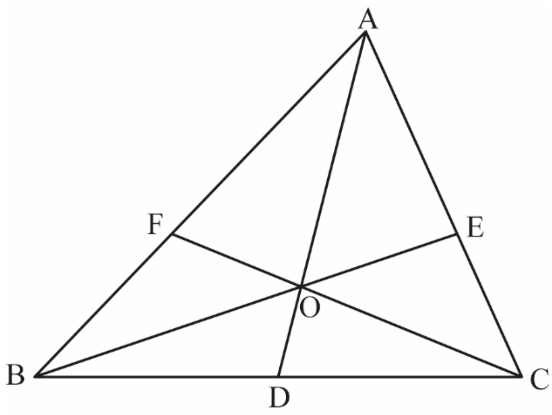
therefore,

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2} < \frac{1}{4}\ln 2 + \frac{1}{2}(1 + \ln 2) = \frac{1}{2} + \frac{3}{4}\ln 2$$ ***A1AG***

[4 marks]

Total [12 marks]

12.



attempting to use Ceva's Theorem **_M1_**

$\frac{\text{AE}}{\text{EC}}\cdot\frac{\text{CD}}{\text{DB}}\cdot\frac{\text{BF}}{\text{FA}}=1 \quad (\text{I})$ **_A1_**

the triangles AFE and ABC are similar **_M1_**

therefore $\frac{\text{AE}}{\text{EC}}=\frac{\text{AF}}{\text{FB}}$ **_A1_**

substituting in (I), $\frac{\text{AF}}{\text{FB}}\cdot\frac{\text{CD}}{\text{DB}}\cdot\frac{\text{BF}}{\text{FA}}=1$ **_M1_**

leading to $\frac{\text{CD}}{\text{DB}}=1$ hence $\text{CD}=\text{DB}$ **A1**

Note: Accept use of Thales' theorem.

[6 marks]

13. (a) (i) use of

$$r=\frac{\sum xy-n\bar{x}\,\bar{y}}{\sqrt{\left(\sum x^2-n\bar{x}^2\right)\left(\sum y^2-n\bar{y}^2\right)}}$$ **_M1_**

$$=\frac{495.4-12\times\frac{76.3}{12}\times\frac{72.2}{12}}{\sqrt{\left(563.7-\frac{12\times76.3^2}{12^2}\right)\left(460.1-\frac{12\times72.2^2}{12^2}\right)}}$$ **_A1_**

$=0.809$ **_A1_**

Note: Accept any answer that rounds to 0.81.

continued…

Question 13 continued

(ii) $t = 0.80856...\sqrt{\dfrac{10}{1-0.80856...^2}}$ **(*M1*)**

$= 4.345\ldots$ ***A1***

p-value $= 7.27\times10^{-4}$ ***A1***

Note: Accept any answer that rounds to 7.2 or 7.3×10^{-4}.
Note: Follow through their p-value

(iii) this value indicates that X, Y are not independent ***A1***

[7 marks]

(b) use of

$y-\bar{y} = \dfrac{\sum xy - n\bar{x}\,\bar{y}}{\sum x^2 - n\bar{x}^2}\left(x-\bar{x}\right)$ ***M1***

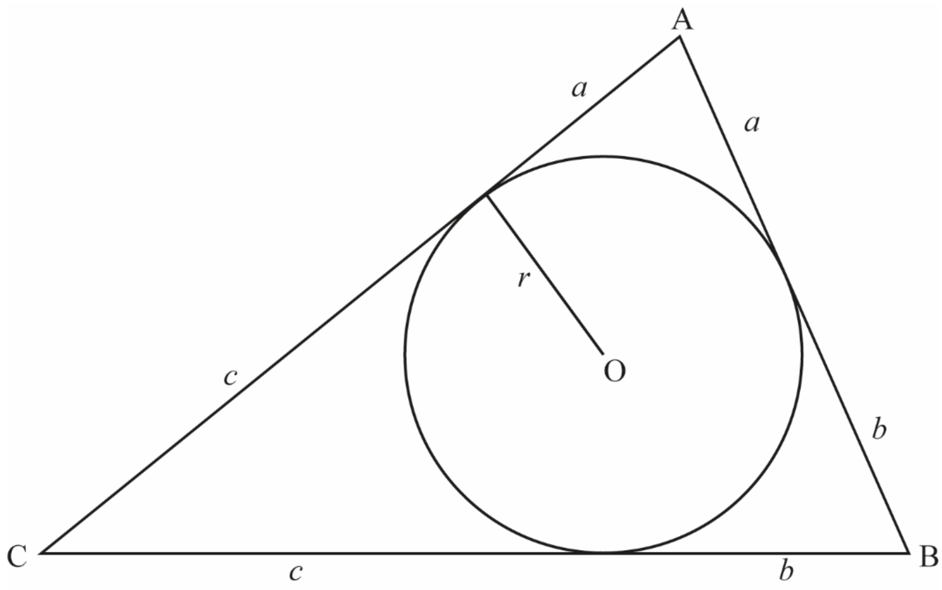
$y-\dfrac{72.2}{12} = \left(\dfrac{495.4-12\times\dfrac{76.3}{12}\times\dfrac{72.2}{12}}{563.7-12\times\dfrac{76.3^2}{12^2}}\right)(x-\dfrac{76.3}{12})$ ***A1***

putting $x = 5.2$ gives $y = 5.5$ ***A1***

[3 marks]

Total [10 marks]

14. (a)



let the lengths of the tangents from A,B,C be a,b,c respectively. Then

$b + c = 12,\ c + a = 10,\ a + b = 8$ ***M1A1***

the solution is $a = 3,\ b = 5,\ c = 7$ ***A1***

[3 marks]

(b) (i) area of triangle $= 3r + 5r + 7r$ ***A1***

$= 15r$ ***AG***

Note: Accept alternative methods.

(ii) $\cos \hat{A} = \dfrac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8}$ ***M1***

$\cos \hat{A} = \dfrac{1}{8}$ ***A1***

$\sin \hat{A} = \sqrt{1 - \dfrac{1}{64}} = \dfrac{\sqrt{63}}{8}$ ***M1***

$= \dfrac{3\sqrt{7}}{8}$ ***AG***

(iii) area of triangle $= \dfrac{1}{2} \times 10 \times 8 \times \dfrac{3\sqrt{7}}{8} = 15\sqrt{7}$ ***M1A1***

$15r = 15\sqrt{7}$ ***M1***

$\Rightarrow r = \sqrt{7}$ ***A1***

$N = 7$

[8 marks]

Total [11 marks]

15. $(1021)_n = n^3 + 2n + 1$ ***A1***

when $n = 3$, $(1021)_3 = 34$ so the proposition is true for $n = 3$ ***A1***

assume the proposition is true for $n = k$, *ie*, $k^3 + 2k + 1$ is not divisible by 3 ***M1***

for $n = k + 1$, consider

$(k+1)^3 + 2(k+1) + 1 = k^3 + 3k^2 + 3k + 1 + 2(k+1) + 1$ ***M1***

$= (k^3 + 2k + 1) + (3k^2 + 3k + 3)$ ***A1***

first term is not divisible by 3 by assumption and second term is divisible by 3, hence the sum of the 2 terms is not divisible by 3 ***R1***

therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 3$, the proposition is true for $n \geq 3$ ***R1***

Note: Only award the final ***R1*** if at least 5 of the previous marks have been awarded.

[7 marks]

16. (a) (i) **METHOD 1**

$\frac{dy}{dx} = \frac{dy/dp}{dx/dp}$ ***M1***

$= \frac{2p}{3p^2}$

$= \frac{2}{3p}$ ***A1***

the equation of the tangent at P is

$y - p^2 + 1 = \frac{2}{3p}(x - p^3)$ ***(M1)(A1)***

this passes through the origin if

$-p^2 + 1 = -\frac{2p^3}{3p}$ ***M1***

$p = \sqrt{3}$ ***A1***

the equation of L is

$y = \frac{2}{3\sqrt{3}}x$ ***A1***

continued…

Question 16 continued

METHOD 2

the point of intersection of the line $y = mx$ and C satisfies

$t^2 - 1 = mt^3$ ***M1***

giving $mt^3 - t^2 + 1 = 0$ ***A1***

the condition for tangency is that this cubic has a double root. The condition for that is that the cubic has a stationary value on the t-axis

consider

$3mt^2 - 2t = 0$ ***M1***

$t = \frac{3}{2m}$ (rejecting $t = 0$) ***A1***

therefore $m\left(\frac{3}{2m}\right)^3 - \left(\frac{3}{2m}\right)^2 + 1 = 0$ ***M1***

$m^2 = \frac{4}{9} - \frac{8}{27} = \frac{4}{27}$ ***A1***

$m = \frac{2}{3\sqrt{3}}$ so the tangent is $y = \frac{2}{3\sqrt{3}}x$ ***A1***

(ii) the coordinates of P are $(3\sqrt{3}, 2)$ ***A1***

[8 marks]

(b) the tangent meets C where

$t^2 - 1 = \frac{2}{3\sqrt{3}}t^3$ ***M1***

$t^3 - \frac{3\sqrt{3}}{2}t^2 + \frac{3\sqrt{3}}{2} = 0$ ***A1***

it is known that this cubic has a double root $t = \sqrt{3}$ so considering the product (or sum) of roots or by factorising ***(M1)***

third root = $-\frac{\sqrt{3}}{2}$ ***A1***

Q is $\left(-\frac{3\sqrt{3}}{8}, -\frac{1}{4}\right)$ ***A1***

[5 marks]

Total [13 marks]